1 Machine - n Jobs Scheduling Algorithm

Problem Domain:

A set of n jobs must be carried out on a single machine that can do only one job at a time. Each job j takes p_j hours to complete. Given job weights w_j for j=1,…..,n in what order should the jobs be carried out so as to minimize the weighted sum of their start times? Formulate this scheduling problem as a mixed integer program.

Facts:

- Given n jobs must be carried on one machine at a time.
- Each job j takes P_j hours to complete and has w_j weight associated with it.
- A need to minimize the weighted sum of their start times.

Jobs (j)

	1	2	3	….	…..	j	….	…	n
1	X								
2		X							
…									
i						X			
….									
n									X

Machining Slots (i)

Traditional Approach

- Introduce the start time variable x_j for job j where x_j>=0
- For every pair of jobs i and j either job i precedes job j or job j precedes job i. Since the machine can only do one job at a time.

Therefore,

$$x_i + p_i \leq x_j$$

$$x_j + p_i \leq x_i$$

Converting the OR into AND constraint

$$x_i - x_j + p_i \leq M(1 - y_{ij})$$

$$x_j - x_i + p_i \leq y_{ij}$$

$$y_{ij} = 0 \quad or \quad 1$$

The final formulation is

$$Objective \quad Function$$

$$\min \sum_{j=1}^{n} w_j x_j$$

$$St$$

$$x_i - x_j + p_i \leq M(1 - y_{ij})$$

$$x_j - x_i + p_i \leq y_{ij}$$

$$y_{ij} = 0 \quad or \quad 1$$

$$where$$

$$M \geq \max\{xi - xj + pi, xj - xi + pi)$$

$$x_i \geq 0, x_j \geq 0, p_i \geq 0$$

New Procedure

- Introduce the start time variable t_i
- First ensure that each job is placed in one column only ones and same with the rows i.e. Some what like the "Assignment Problem"
- To add constraint for this, introduce decision variable y_{ij} ,where y_{ij} =1 if selected else y_{ij} =0
- To take care of processing time and start times of job use

Start Time of Job j + Processing Time for Job j =Start Time for Job j+1

- Start Time of 1st Job is 0
- Formulation of Objective Function(Non-Linear Function)
- Addition of constraint to give the lower limits for variables.

Advantages:

	New Approach	Traditional Approach
# of Variables	$n*n+n=n^2+n$	$n*n+n=n^2+n$
# of Constraints	n+n+n=3n	2*(n!-n)

As number of constraints are reduced considerably, the new approach will be faster.

Objective Function

$$Min \sum_{i=1}^{n} t_i \sum_{j=1}^{n} w_j y_{ij}$$

st

$$ti+1 = \sum_{j=1}^{n} p_j y_{ij} + t_i \quad for\ i = 1\ to\ n\text{-}1$$

$$\sum_{j=1}^{n} y_{ij} = 1 \qquad for\ i = 1\ to\ n$$

$$\sum_{i=1}^{n} y_{ij} = 1 \qquad for\ j = 1\ to\ n$$

$$t_1 = 0$$

$$ti \geq 0, wj \geq 0\ and\ y_{ij} = \{0, 1\}$$